Unification theory of angular magnetoresistance oscillations in quasi-one-dimensional conductors

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We present a unification theory of angular magnetoresistance oscillations, experimentally observed in quasi-one-dimensional organic conductors, by solving the Boltzmann kinetic equation in the extended Brillouin zone. We find that, at commensurate directions of a magnetic field, resistivity exhibits strong minima. In two limiting cases, our general solution reduces to the results, previously obtained for the Lebed Magic Angles and Lee-Naughton-Lebed oscillations. We demonstrate that our theoretical results are in good qualitative and quantitative agreement with the existing measurements of resistivity in $(TMTSF)_2ClO_4$ conductor.

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I. INTRODUCTION

Magnetic properties of quasi-one-dimensional (Q1D) organic conductors have been intensively studied both experimentally and theoretically since a discovery of the so-called field-induced spin-density-wave (FISDW) phase diagrams.[1,2] For open electron orbits, where the Landau-level quantization is impossible, as theoretically shown,[1,3–5] other quantum effects—the Bragg reflections of electrons from the Brillouin zones—play an important role. In the simplest situation, where magnetic field is perpendicular to conducting layers of Q1D conductors, the Bragg reflections are demonstrated[3–5] to be responsible for the FISDW phases formation. In an inclined magnetic field, perpendicular to conducting chains, a more complicated interference picture of electrons, moving in the extended Brillouin zones, appears. As shown in Refs. 6 and 7, it results in the constructive interference of electron waves in some many-body effects for some special commensurate directions of a magnetic field, which are called the Lebed magic angles (LMAs).

The LMA effects were experimentally discovered in Refs. 8–13 and are observed in a number of organic conductors,[14–29] which possess Q1D parts in their Fermi surfaces (FSs). Note that, instead of maxima of resistivity due to electron-electron scattering, predicted in Ref. 7, the experiments[8–29] demonstrate clear minima at the LMA directions of a magnetic field. In important theoretical contributions,[30,31] it was shown that constructive interference effects[6,7] could appear in such one-body phenomenon as a residual resistivity due to impurities. As a result, minima of resistivity component, perpendicular to conducting layers, were theoretically found at the LMA directions of a magnetic field.[30,31] Nevertheless, theoretical model[30] predicted weak (i.e., exponentially small) magnitudes of the LMA minima whereas Ref. 31 was based on an incorrect solution of the Boltzmann kinetic equation.

Recently, a correct solution of the Boltzmann equation for a magnetic field, perpendicular to conducting chains of a Q1D conductor, was found and the existence of the strong LMA minima in perpendicular to conducting layers component of resistivity was firmly established.[32] In addition, a quantum-mechanical variant of the theory was suggested.[33] Theory[33] reveals quantum interference nature of periodic solutions of the Boltzmann kinetic equation in the extended

Brillouin zone,[32] which lead to the appearance of the LMA effects. According to Ref. 33, the LMA minima of resistivity appear due to changes in electron wave functions dimensionality from 1D into two-dimensional (2D) at commensurate directions of a magnetic field[6,7] due to constructive interference effects. These interference effects appear between velocity component, perpendicular to conducting layers, and the density of states.[33] Note that the theory,[32,33] which is a limiting case of the unification theory, suggested in this paper, can explain the experimental observations of the LMA effects only in resistivity.[8–29] As to the observations of anomalously strong LMA phenomena in the Nernst[34–38] and Hall[39] effects, their explanations may need a different theoretical approach.

As experimentally discovered,[40–43] the LMA-like magnetoresistance minima with enhanced magnitudes are observed[40–46] in experimental geometry, where magnetic field is inclined with respect to conducting chains of a Q1D conductor. At first, these angular oscillations were interpreted[42,43] in terms of the LMA phenomenon. Later, it became clear[47–50] that, although they are related to the LMA effects, their concrete physical meaning is quite different. Below, we call the above mentioned angular oscillations of resistivity the Lee-Naughton-Lebed (LNL) ones.

Theory of the LNL phenomenon, based on a solution of the Boltzmann kinetic equation in the extended Brillouin zone, was suggested in Refs. 47 and 48. In Refs. 47 and 49, theory of the LNL oscillations was extended to the so-called weak non-Fermi-liquid quantum case.[47] In Refs. 44, 45, and 50, the quantum theory of the LNL oscillations was suggested for the Fermi-liquid case and their quantum interference nature was revealed. According to Refs. 44, 45, and 50, the LNL angular oscillations are due to changes in electron wave functions dimensionality from 1D into 2D at some commensurate directions of a magnetic field due to constructive interference effects. In contrast to case of the LMA oscillations, these interference effects appear between two velocity components, perpendicular to the conducting chains. By present time, the physical origin of the LNL oscillations has been firmly established[44–50] and a comparison of the theoretical results with the existing experimental data have been made.[44,45,50] These include theories of the LNL phenomenon in the presence of anion ordering potentials[44,45,51] and theory,[52] connecting the LNL oscillations with the Aharonov-Bohm effect.
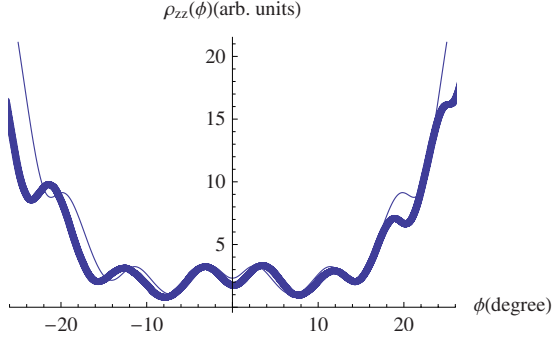
FIG. 1. (Color online) Comparison of the theory (12) and the experiment (Ref. 57) for the LNL oscillations in (TMTSF)$_2$ClO$_4$. Thin solid curve is calculated from Eq. (12) at $\theta = 7°$, with band parameters being $t_a/t_b = 8.5$ and $\Delta/2t_b = 0.1$. For calculations, we have used $\omega_b(0)\tau = 15$ and $c^* = 2b^*$. Numerical results are in good qualitative and quantitative agreement with the experimental data (thick solid line) in a broad range of angle ϕ.

The goal of our paper is to suggest an analytical unified theory, which describes both the LMA and LNL phenomena as its limiting cases. Note that the suggested theory also describes the so-called Danner-Kang-Chaikin oscillations[53] and third angular effect.[54–56] To derive an analytical expression for resistivity component, perpendicular to conducting layers, we analytically solve the Boltzmann kinetic equation in the extended Brillouin zone in a magnetic field, inclined with respect to conducting chains of a Q1D conductor. We find strong minima of resistivity, corresponding to the appearance of periodic solutions of the Boltzmann equation at the LMA and LNL commensurate directions of a magnetic field. A comparison of our theoretical results with the experimental data on (TMTSF)$_2$ClO$_4$ conductor[57] shows good qualitative and quantitative agreement (see Figs. 1 and 2).
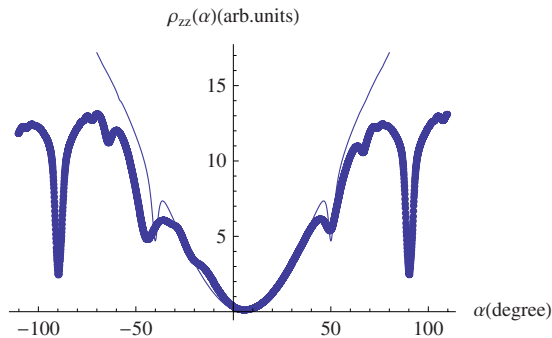

FIG. 2. (Color online) Comparison of the theory (26) and experiment (Ref. 57) for the LMA phenomena in (TMTSF)$_2$ClO$_4$, where $g(p_y b^*) = [\cos^2(p_y b^*) + (\Delta/2t_b)^2]^{1/2}$ in Eq. (1). Thin solid curve is numerically calculated from Eq. (26), with band parameters $t_a/t_b = 8.5$ and $\Delta/2t_b = 0.1$. For the calculations, we have used $\omega_b(0)\tau = 15$ and $c^* = 2b^*$. There exist good qualitative and quantitative agreement between the theory and experiment for $0° < \alpha < 60°$. For negative values of α, agreement between the theory and experiment is worse, perhaps, due to some problems with the experimental measurements.

II. UNIFICATION THEORY

Let us consider a Q1D conductor with the following electron spectrum:

$$\varepsilon^{\pm}(\mathbf{p}) = \pm v_x(p_y b^*)[p_x \mp p_x(p_y)] - 2t_c \cos(p_z c^*),$$

$$p_x(p_y) = p_F - 2t_b g(p_y b^*)/v_F. \qquad (1)$$

[Here, $+(-)$ stands for right (left) sheet of the FS; v_F and p_F are the Fermi velocity and Fermi momentum along the most conducting $\mathbf{x}$ axis, respectively; t_b and t_c are the hopping integrals along $\mathbf{y}$ and $\mathbf{z}$ axes.] Note that in Eq. (1), in accordance with Refs. 31–33, p_y dependence of the velocity along conducting chains, $v_x(p_y)$, is taken into account. For most Q1D conductors, we can choose $g(p_y b^*) = \cos(p_y b^*)$ in Eq. (1) but for an important exception of (TMTSF)$_2$ClO$_4$ conductor with an anion ordering, we have to use $g(p_y b^*) = [\cos^2(p_y b^*) + (\Delta/2t_b)^2]^{1/2}$, where Δ is the the so-called anion gap.[1,2,44,45]

Below, we show that, when a Q1D conductor [Eq. (1)] is placed in a tilted magnetic field,

$$\mathbf{H} = H(\cos\theta\cos\varphi, \cos\theta\sin\varphi, \sin\theta), \qquad (2)$$

and a weak electric field perpendicular to the conducting $(\mathbf{x}, \mathbf{y})$ plane, then at certain orientations of the field,

$$\sin\varphi = n\left(\frac{b^*}{c^*}\right)\tan\theta, \qquad (3)$$

where n is an integer, resistivity, $\rho_{zz}(H, \theta, \varphi)$, exhibits strong minima.

The Boltzmann kinetic equation in crossed electric and magnetic fields in the so-called τ approximation can be written in a standard way,

$$\left[e\mathbf{E} + \left(\frac{e}{c}\right)[\mathbf{v} \times \mathbf{H}]\right]\frac{\partial f(\mathbf{p})}{\partial \mathbf{p}} = -\frac{f(\mathbf{p}) - f_0(\mathbf{p})}{\tau}. \qquad (4)$$

[In Eq. (4), $f(\mathbf{p})$ is an electron distribution function and τ is a relaxation time.] After the standard approximation,

$$f(\mathbf{p}) = f_0(\mathbf{p}) - \frac{\partial f_0(\mathbf{p})}{\partial \varepsilon}\Psi(p_y, p_z), \qquad (5)$$

the Boltzmann Eq. (4) can be written as[58]

$$e\mathbf{E} \cdot \mathbf{v} - \left(\frac{e}{c}\right)[\mathbf{v} \times \mathbf{H}] \cdot \frac{\partial \Psi(p_y, p_z)}{\partial \mathbf{p}} = \frac{\Psi(p_y, p_z)}{\tau}, \qquad (6)$$

where $f_0(\mathbf{p})$ is the Fermi-Dirac distribution function. [Note that, in Eq. (6), we use independent variables (ϵ, p_y, p_z),[58] instead of (p_x, p_y, p_z), where energy ϵ is conserved in the absence of electric field. Since both electric field and temperature are supposed to be small, the electric conductivity is defined by electrons, located in the near vicinity of the Q1D FS. Therefore, the distribution function $\Psi(p_y, p_z)$ can be taken at $\epsilon = \epsilon_F$ and, thus, does not depend on energy in Eq. (6).] Taking into account that $\mathbf{v} = \partial\varepsilon(\mathbf{p})/\partial\mathbf{p}$, we can rewrite the Boltzmann equation as

$$eEv_z^0 \sin z - \omega_b(y,\theta)\frac{\partial \Psi(y,z)}{\partial y} - \omega_c(y,\theta,\varphi)\frac{\partial \Psi(y,z)}{\partial z}$$

$$- \omega_c^*(\theta,\varphi)g'(y)\frac{\partial \Psi(y,z)}{\partial z} = \frac{\Psi(y,z)}{\tau}. \quad (7)$$

To simplify the notations, we define the following dimensionless parameters: $y=p_y b^*$, $z=p_z c^*$, and the following frequency variables:

$$\omega_b(y,\theta) = \left(\frac{e}{c}\right)v_x(y)Hb^* \sin \theta,$$

$$\omega_c(y,\theta,\varphi) = \left(\frac{e}{c}\right)v_x(y)Hc^* \cos \theta \sin \varphi,$$

$$\omega_c^*(\theta,\varphi) = \left(\frac{e}{c}\right)v_y^0 Hc^* \cos \theta \cos \varphi, \quad (8)$$

where $v_y^0=2t_b b^*$ and $v_z^0=2t_c c^*$; $\hbar=1$.

It is important that the partial differential Eq. (7) can be analytically solved (see the Appendix),

$$\Psi(y,z) = eEv_z^0 \int_{-\infty}^0 \frac{du}{\omega_b(y+u,\theta)}\sin\left\{z - \int_u^0 \frac{dv}{\omega_b(y+v,\theta)}\right.$$

$$\times[\omega_c(y+v,\theta,\varphi) + \omega_c^*(\theta,\varphi)g'(y+v)]\Big\}$$

$$\times\exp\left[-\int_u^0 \frac{dv}{\tau\omega_b(y+v,\theta)}\right]. \quad (9)$$

Since the solution (9) of the Boltzmann Eq. (7) is known, we can express electric current density along **z** axis, perpendicular to conducting layers, in the following way:

$$j_z \sim \int dy \int dz v_z^0 \sin z \frac{\partial f_0(\mathbf{p})}{\partial \varepsilon}\Psi(y,z). \quad (10)$$

In Eq. (10), we omit an exact factor since conductivity in a magnetic field is scaled below to its value in the absence of the field. Notice that $\partial f_0(\mathbf{p})/\partial \varepsilon$ in Eq. (10) is the density of states at the FS, which is proportional to $1/v_x(y)$. From Eqs. (9) and (10), we obtain the following result for interlayer conductivity after some calculations:

$$\sigma_{zz}(H,\theta,\varphi) \sim \int \frac{dy}{v_x(y)}\int_{-\infty}^0 \frac{du}{\omega_b(y+u)}\cos\left\{\int_u^0 \frac{dv}{\omega_b(y+v,\theta)}\right.$$

$$\times[\omega_c(y+v,\theta,\varphi) + \omega_c^*(\theta,\varphi)g'(y+v)]\Big\}$$

$$\times\exp\left[-\int_u^0 \frac{dv}{\omega_b(y+v)\tau}\right]. \quad (11)$$

[Note that, in a Q1D case resistivity component, perpendicular to conducting layers, is $\rho_{zz}(H,\theta,\varphi)=1/\sigma_{zz}(H,\theta,\varphi)$.] Eqs. (9)–(11) and comparison of Eq. (11) limiting cases with the experimental data, obtained on (TMTSF)$_2$ClO$_4$ conductor,[57] are the main results of our paper.

III. LIMITING CASES

In this section, we consider two different experimental settings, which correspond to the LNL and LMA phenomena, repetively.

A. LNL oscillations

In experiments, where the LNL oscillations are observed, magnetic field is tilted away from $(\mathbf{y},\mathbf{z})$ plane. In the case, where $\theta \ll 90°$, the p_y dependence of the Fermi velocity is not significant, and, therefore, we can everywhere replace $v_x(p_y)$ into v_F. This significantly simplifies the formula for conductivity [Eq. (11)],

$$\sigma_{zz}(H,\theta,\varphi) \sim \int_{-\infty}^0 dz \exp(z)\int_0^{2\pi} \frac{dy}{2\pi}\cos\left(\omega_c(\theta,\varphi)\tau z\right.$$

$$+ \frac{\omega_c^*(\theta,\varphi)}{\omega_b(\theta)}\{g(y) - g[y + \omega_b(\theta)\tau z]\}\right), \quad (12)$$

where the frequency variables are defined in Eq. (8) with $v_x(y)=v_F$. The existence of resistivity minima at the commensurate directions of a magnetic field [Eq. (3)] can be understood from Eq. (12). Since $g(y)$ is a periodic function of y, at the commensurate directions [where a condition $\omega_c(\theta,\varphi)=n\omega_b(\theta)$ is satisfied], constructive interference effects increase the integral [Eq. (12)], which gives the minima in $\rho_{zz}(H,\theta,\varphi)$.

In Fig. 1, we show both the theoretical results, obtained by means of numerical calculations of Eq. (12), and the experimental data,[57] obtained on (TMTSF)$_2$ClO$_4$ conductor. Note that the minima of resistivity appear only for even values of n in Eq. (3) due to the existence of the anion ordering gap, Δ, in (TMTSF)$_2$ClO$_4$ electron spectrum, $g(p_y b^*) = [\cos^2(p_y b^*)+(\Delta/2t_b)^2]^{1/2}$ [see Eq. (1)].

Most Q1D conductors [e.g., (TMTSF)$_2$PF$_6$ and $\kappa-$(ET)$_2$Cu(NCS)$_2$] can be described by the electronic spectrum [Eq. (1)] with $g(y)=\cos y$. In this case, Eq. (12) reduces to

$$\sigma_{zz}(H,\theta,\varphi) \sim \int_{-\infty}^0 dz \exp(z)\int_0^{2\pi} \frac{dy}{2\pi}\cos\left[\omega_c(\theta,\varphi)\tau z\right.$$

$$+ \frac{\omega_c^*(\theta,\varphi)}{\omega_b(\theta)}\{\cos(y) - \cos[y + \omega_b(\theta)\tau z]\}\right]. \quad (13)$$

Using the identity,

$$\exp(iz \cos \theta) = \sum_{n=-\infty}^{+\infty} J_n(z)i^n e^{in\theta}, \quad (14)$$

we can further simplify Eq. (13),

$$\sigma_{zz}(H,\theta,\varphi) \sim \int_{-\infty}^0 dz \exp(z)\sum_{n=-\infty}^{+\infty} J_n^2\left[\frac{\omega_c^*(\theta,\varphi)}{\omega_b(\theta)}\right]$$

$$\times\cos\{[\omega_c(\theta,\varphi) - n\omega_b(\theta)]\tau z\}. \quad (15)$$

Finally, Eq. (15) can be transformed into a simple analytical form,

$$\frac{\sigma_{zz}(H,\theta,\varphi)}{\sigma_{zz}(H=0)} = \sum_{n=-\infty}^{\infty} \frac{J_n^2[\omega_c^*(\theta,\varphi)/\omega_b(\theta)]}{1+\tau^2[\omega_c(\theta,\varphi)-n\omega_b(\theta)]^2}, \quad (16)$$

where $J_n(\cdots)$ is the nth-order Bessel function. The oscillatory behavior of interlayer resistivity is directly seen from Eq. (16). Indeed, at the commensurate directions of a magnetic field [Eq. (3)], where $\omega_c(\theta,\varphi)=n\omega_b(\theta)$, there appear maxima of conductivity [Eq. (16)], which lead to resistivity minima in $\rho_{zz}(H,\theta,\varphi)$ (see Fig. 1). Note that Eq. (16) is an agreement with the previous results (Refs. 47–50).

B. LMA effects

The LMA phenomena are experimentally observed in a magnetic field, directed in $(\mathbf{y},\mathbf{z})$ plane,

$$\mathbf{H} = (0, H\sin\alpha, H\cos\alpha), \quad (17)$$

where $\alpha=90°-\theta$. Therefore, Eq. (3) for the commensurate directions of a magnetic field reduces to

$$\tan\alpha = n\left(\frac{b^*}{c^*}\right), \quad (18)$$

where n is integer. In this case, where $\phi=90°$, Eq. (11) for interlayer conductivity can be rewritten as

$$\sigma_{zz}(H,\alpha) \sim \int \frac{dy}{v_x(y)} \int_{-\infty}^{0} du \frac{\cos[N(\alpha)u]}{\omega_b(y+u,\alpha)}$$
$$\times \exp\left[-\int_u^0 \frac{dv}{\tau\omega_b(y+v,\alpha)}\right], \quad (19)$$

where $\omega_b(y,\alpha)=eHv_x(y)b^*\cos\alpha/c$, $\omega_c(y,\alpha)=eHv_x(y)c^*\sin\alpha/c$, and $N(\alpha)=\omega_c(y,\alpha)/\omega_b(y,\alpha)$. Equation (19) can be transformed to the following expression by integrating by parts:

$$\sigma_{zz}(H,\alpha) \sim \int \frac{dy}{v_x(y)}\left\{1+N(\alpha)\int_{-\infty}^{0} du \, \sin[N(\alpha)u] \right.$$
$$\left. \times \exp\left[-\int_u^0 \frac{dv}{\tau\omega_b(y+v,\alpha)}\right]\right\}. \quad (20)$$

If we introduce the following notations:

$$f(y) = \frac{v_F}{v_x(y)} - 1,$$

$$h_b(H,\alpha) = \frac{e}{c}Hv_Fb^*\tau\cos\alpha,$$

$$h_c(H,\alpha) = \frac{e}{c}Hv_Fc^*\tau\sin\alpha, \quad (21)$$

where $1/v_F=\langle 1/v_x(y)\rangle_{p_y}$, then Eq. (20) becomes,

$$\sigma_{zz}(\theta) \sim \int dy[1+f(y)][1+h_c(H,\alpha)\int_{-\infty}^{0} du \, \sin[h_c(H,\alpha)u]$$

$$\times \exp[-\int_u^0 dv\{1+f[y+h_b(H,\alpha)v]\}]]. \quad (22)$$

Integrating by parts one more time and taking into account a periodicity of function $f(y)$, we obtain the following expression for conductivity:

$$\frac{\sigma_{zz}(H,\theta)}{\sigma_{zz}(H=0)} = 1 - h_c^2(H,\alpha)\int_{-\infty}^{0} du \, \exp(u)$$

$$\times \cos[h_c(H,\alpha)u]\int_0^{2\pi} \frac{dy}{2\pi}$$

$$\times \exp\{-\int_u^0 du_1 f[y+h_b(H,\alpha)u_1]\}. \quad (23)$$

In the so-called clean limit, where $h_b(H,\alpha)\gg 1$, Eq. (23) can be significantly simplified. Below, we introduce the Fourier transform of function $f(y)$,

$$f(y) = \sum_{n=1}^{+\infty} A_n \cos(ny). \quad (24)$$

In the clean limit, the last exponential function in Eq. (23), whose argument is inversely proportional to $h_b(H)$, can be expanded as

$$\exp\{-\int_u^0 du_1 f[y+h_b(H,\alpha)u_1]\}$$

$$= 1 - \int_u^0 du_1 \sum_{n=1}^{+\infty} A_n \cos\{n[y+h_b(H,\alpha)u_1]\}$$

$$+ \frac{1}{2}\int_u^0 du_1 \int_u^0 du_2 \sum_{n,m=1}^{+\infty} A_nA_m \cos\{n[y+h_b(H,\alpha)u_1]\}$$

$$\times \cos\{m[y+h_b(H,\alpha)u_2]\}, \quad (25)$$

where the higher-order terms are discarded. After integration with respect to variable y, the second term in Eq. (25) vanishes, whereas, in the third term, only contributions with $n=m$ retain. Finally, the interlayer conductivity can be represented as

$$\frac{\sigma_{zz}(H,\alpha)}{\sigma_{zz}(H=0)} = \frac{1}{1+h_c^2(H,\alpha)} - \tan^2\alpha\left(\frac{c^*}{2b^*}\right)^2$$

$$\times \sum_{n=1}^{\infty} \frac{A_n^2}{n^2}\left(\frac{2}{1+h_c^2(H,\alpha)}\right.$$

$$- \frac{1}{1+[h_c(H,\alpha)-nh_b(H,\alpha)]^2}$$

$$\left. - \frac{1}{1+[h_c(H,\alpha)+nh_b(H,\alpha)]^2}\right). \quad (26)$$

Note that Eq. (26) is an agreement with our previous results (Refs. 32 and 33).

In Fig. 2, we compare the theory (26) with the experimental data[57] for the LMA phenomenon. It is important that, for the calculations, we have used in Eq. (26) the same values of the parameters as for the calculations of the LNL phenomenon [see Eq. (12) and Fig. 1]. Figure 2 demonstrates good qualitative and quantitative agreement between the theory and experiment in the wide range of the angles: $0° < \alpha < 60°$. For $|\alpha| > 60°$, there appear significant deviations from the experimental behavior.[57] One possible reason for that is a breakdown of Eq. (26) for large values of angle α, where the so-called clean-limit approximation is not valid. Another possible reason for the deviations is that, at high values of angle α (i.e., at high in-plane projections of a magnetic field), there may occur Fermi-liquid[59] or non-Fermi-liquid[16] decoupling of the conducting layers, where the Boltzmann kinetic equation is not valid any more.

IV. CONCLUSION

In this work, we propose a unification theory for angular magnetoresistance oscillations, experimentally observed in Q1D organic conductors. We analytically solve the Boltzmann equation in the extended Brillouin zone and find analytical formula, which describes interlayer resistivity. We show that, in two important limiting cases, this formula reduces to the expressions, previously obtained to describe the LNL and LMA phenomena in resistivity. Numerical results, obtained from these expressions, are shown to be in good agreement with the experimental data, obtained on $(TMTSF)_2ClO_4$ conductor, in a broad rage of magnetic field directions. On the other hand, a comparison of the theory with the LMA experimental data reveals significant discrepancy between the theory and experiment for directions of a magnetic field close to the conducting layers. We suggest that this discrepancy may be due to decoupling of the conducting layers in a parallel magnetic field.

APPENDIX: PROOF OF Eq. (9)

From physical point of view, we have to find a particular solution of the inhomogeneous partial differential Eq. (7), which is proportional to external electric field and periodic in the extended Brillouin zone with respect to both variables, y

and z. To find such solution, below we use the method of characteristics for the first-order partial differential Eq. (7). Let us consider the following equation, which defines the characteristics:

$$\frac{dz}{dy} = \frac{\omega_c(y,\theta,\varphi) + \omega_c^*(\theta,\varphi)g'(y)}{\omega_b(y,\theta)} \quad \text{(A1)}$$

with initial condition being,

$$z(u) = z_0. \quad \text{(A2)}$$

Note that Eqs. (A1) and (A2) have the following solution:

$$z(y) = z_0 + \int_u^y \frac{dv}{\omega_b(v,\theta)}[\omega_c(v,\theta,\varphi) + \omega_c^*(\theta,\varphi)g'(v)]. \quad \text{(A3)}$$

At this point, Eq. (7) can be considered as an ordinary differential equation,

$$eEv_z^0 \sin z_0 - \omega_b(y,\theta)\frac{d\Psi(y,z_0)}{dy} = \frac{\Psi(y,z_0)}{\tau}. \quad \text{(A4)}$$

For our purpose, we choose the following solution of Eq. (A4):

$$\Psi(y,z_0) = eEv_z^0 \int_{-\infty}^y \frac{dv}{\omega_b(v,\theta)}\sin z_0 \exp\left\{ -\int_v^y \frac{dw}{\omega_b(w,\theta)\tau} \right\}. \quad \text{(A5)}$$

If we take into account that z_0 is given by Eq. (A3), we can obtain

$$\Psi(y,z) = eEv_z^0 \int_{-\infty}^y \frac{du}{\omega_b(u,\theta)}$$
$$\times \sin\left[z - \int_u^y \frac{dv}{\omega_b(v,\theta)}[\omega_c(v,\theta,\varphi) + \omega_c^*(\theta,\varphi)g'(v)] \right]$$
$$\times \exp\left[-\int_u^y \frac{dv}{\tau\omega_b(v,\theta)} \right]. \quad \text{(A6)}$$

From Eq. (A6), it is directly seen that the obtained solution of Eq. (7) is proportional to electric field and periodic with respect to variable z. It is possible to make sure that it is also periodic with respect to variable y. For this purpose, it is necessary to take into account that the functions $\omega_b(y,\theta)$, $\omega_c(y,\theta,\phi)$, and $g(y)$ are periodic with respect to variable y in the extended Brillouin zone. After changes in integration variables, $u \to y+u$ and $v \to y+v$, we can finally obtain Eq. (9) from Eq. (A6).

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